2023 Team Math Attack Contest

Team Contest

December 9, 2023

Rules

1. You have 60 minutes to complete 20 problems (3 minutes/problem).

2. You start at 30 points, and every problem is worth 6 points (max score is 150).

3. You lose 1.5 points for every problem answered incorrectly, and get 0 points for every unanswered problem.

4. NO CALCULATORS. You will be disqualified if you use one.

5. EXACT VALUES ONLY (we want numbers like $\sqrt{2}$ and π)

6. Have fun and think hard!

Team Problems

1. What is the value of $2 \times 2 + 0 \times 0 + 2 \times 2 + 3 \times 3$?

2. Anna sells 7 cookies for 5 dollars. How many dollars will Anna get for selling 28 cookies?

3. A square's side length is randomly chosen from 1,2,3,4,5,6. What is the probability that the square's perimeter is more than 10?

4. Three different positive integers multiply to 77. What is their sum?

5. In an election, Donald received 60% of the votes and Barry received the rest. If Donald won by 24 votes, how many people voted?

6. Joey flips a coin 5 times. What is the probability that the coin came up heads 3 times in a row, and only came up heads 3 times?

7. The decimal expansion of $\frac{1}{7}$ is $0.\overline{142859}$. What is the 2023rd digit in the decimal expansion of $\frac{1}{7}$ (The first digit is 1?

8. Let a be the area of the smallest circle containing a square with side length 1. Let b be the area of the largest circle contained in a square with side lengths 1. What is $a - b$?

9. If 20, 23, and x form an arithmetic sequence, what is the sum of all possible values of x? An arithmetic sequence is where each consecutive terms have a common difference (ex. $2, 4, 6$ is an arithmetic sequence since $4 - 2 = 6 - 4 = 2$).

10. The Math Attack Society consists of Erik, his besties, and several other members. Erik is too tired to make 300 problems for the contest on his own, so he enlists the help of his 4 besties, who make problems at the same speed he does. If he works together with his besties, he will finish in 20 days. The remaining MAS members are not as effective as Erik and his besties are: each of them can only make one problem per day. If he has every other member of Math Attack Society in addition to his besties, he can finish within 15 days. How many people are in Math Attack Society (including him and his besties)?

11. On a rainy day, Jack has a 40% chance of going to the park. Otherwise, he has a 50% chance of going to the park. The probability that it rains on any given day is 60%. One day, Jack went to the park. What is the chance that it rained on that day?

12. If $x = 2^0 + 2^1 + 2^2 + ... + 2^{2023}$, what is the units digit of x?

13. A palindrome is a number that is read the same backwards and forwards (e.g 12321). How many 4 digit palindromes are divisible by 12?

14. If two of the altitudes of an acute triangle divide it as shown in the following diagram, what is the value of x?
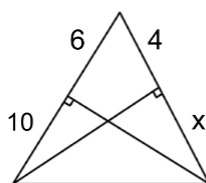
15. Jesse is choosing courses at Squidward Community College. He may select from Physics, Chemistry, Biology, Math, English, History, Art, and Politics. Jesse must choose 5 courses, but he does not want to take all 3 sciences (Physics, Chemistry, Biology). How many ways can Jesse select his courses?

16. Let there be two circles, one with center A with radius 5, the other with center B with radius 12. The distance between A and B is 13, and the two circles intersect at points C and D. What is the length of CD?

17. Cindy bakes some cookies, and arranges them in a rectangular array. If she removes 1 row of the cookies, she is left with 168 cookies. If she instead removes 1 column of cookies, she is left with 165 cookies. How many cookies did she have at the beginning?

18. There is a rectangle in the coordinate plane with vertices at $(0,0)$, $(0,2023)$, $(2,0)$, and $(2,2023)$. A point is chosen randomly in this rectangle. What is the probability that the point is within $1/(2\sqrt{\pi})$ units of distance from a lattice point (a lattice point is a point with integer coordinates)? Express your answer as a simplified fraction.

19. In triangle ABC, AM is the median of side BC. Point E is on side AB, and F is the intersection between CE and AM such that CF/EF = 3/2. What is the ratio of AE/BE?

20. 10^{20} has 441 factors. How many of those factors are strictly in between 10^4 and 10^{10} exclusive (e.g. $2^7 \times 5^3 = 16000, 10^4 < 16000 < 10^{10}$)?