2023 Team Math Attack Contest

Relay Round

December 9, 2023

Rules

1. You have 60 minutes to complete 5 problems with 3 parts (A, B, C) each (4 minutes/part).

2. You get 6 points for Part A, 8 points for Part B, and 10 points for Part C

3. You start at 30 points, and every problem is worth 24 points (max score is 150).

4. You lose 1.5 points for every problem answered incorrectly, and get 0 points for every unanswered problem.

5. NO CALCULATORS. You will be disqualified if you use one.

6. EXACT VALUES ONLY (we want numbers like $\sqrt{2}$ and π)

7. Have fun and think hard!

Problem 1.1

$2^x = 4^y$

$x + 3y = 10$

Find $P = x + y$.

Problem 1.2

Let P be the answer to the previous problem. There are X ways that P different people can stand in a circle. Two ways are considered identical if they can be rotated to match each other. Find X.

Problem 1.3

Let X be the answer to the previous problem. The sum of the factors of X is M, and the number of factors of X is N. Find $M + N$.

Problem 2.1

A soccer player has successfully scored 16 goals out of 29 attempts. Let X be the additional goals he has to score in order for his success rate to be 90%. What is the remainder when X is divided by 100? Let this be S.

Problem 2.2

Let S be the answer to the previous problem. Aiden writes one contest problem every 12 hours. Edward writes S contest problems every hour. If Aiden and Edward work together, how many hours T does it take to make 1 problem?

Problem 2.3

Let T be the answer to the previous problem. Water drains out of a tank at a rate of T liters per second. The tank drains for 26 minutes, before a new pipe is added to the tank, providing water at a rate of 0.5 liters per second. The tank drains for 52 more minutes before it is empty. How much water did the tank have at the beginning?

Problem 3.1

There is a very sad pet store where they sell cats. On the first day, half their cats run away and then they sell one cat. On the second day, a third of the remaining cats run away and they then sell 2 cats. On the third day, 1/4 of their remaining cats run away, but they then sell 5 cats. Now the only cat remaining is Mittens. How many cats did they have at the start?

Problem 3.2

Let m be the answer to the previous problem. How many 3 digit numbers have a digit sum equal to $m/2$?

Problem 3.3

Let n be the answer to the previous problem. Jason writes every number from 1 to $n/2$. He then randomly chooses 2 distinct numbers from the list and multiplies them together. What is the probability that the result will be divisible by $n/2$?

Problem 4.1

John is thinking of a two digit prime number that is one more than a perfect square. What is the largest number he could be thinking of?

Problem 4.2

Let x be the answer to the previous problem. If 12 , 35, and x are the side lengths of a triangle, what is the height of the triangle for the side of length x? (in terms of $\frac{m}{n}$, where m and n share no common factors)

Problem 4.3

Let $p = m - 5n$. How many positive integers less than p share no common factors (other than 1) with p?

Problem 5.1

When x is divided by 7, the remainder is 2. What is the remainder when $4x$ is divided by 7?

Problem 5.2

Let y be the answer to the previous problem. A certain trapezoid has side lengths 7, 5, y, and 5 in that order. What is the product of the lengths of its diagonals?

Problem 5.3

Let z be the answer to the previous problem. A certain polynomial is in the form $f(x) = x^3 + Ax^2 + Bx + (z + 8)$. If this polynomial has three distinct integer roots, what is the sum of all possible values of A?